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HONORS CALC II F 08, MIDTERM I

(1) Find the derivative of the function $\int_{\sqrt{x}}^{x} \frac{e^t}{t} dt$.

(Suggestion: Write the integral as a difference between two integrals, one from 0 to x and another from 0 to $\sqrt{x}$.

Write $f(t) = \frac{e^t}{t}$. Set

$$I(x) = \int_{\sqrt{x}}^{x} f(t)dt = I_1(x) - I_2(x), \text{ where } I_1(x) = \int_0^x f(t)dt, I_2(t) = \int_0^{\sqrt{x}} f(t)dt.$$

Hence by FTC and the chain rule,

$$I'(x) = f(x) - \frac{f(\sqrt{x})}{2\sqrt{x}} = \frac{e^x}{x} - \frac{e^{\sqrt{x}}}{2x} = \frac{2e^x - e^{\sqrt{x}}}{2x}.$$

(2) Evaluate the integral

$$\int \frac{dx}{x^2\sqrt{x^2-16}}.$$

Change $x = 4\sec t$. Then $dx = 4\tan t \sec t dt$ and $x^2 - 16 = 16\tan^2 t$. So

$$I = \int \frac{\tan t \sec t}{16\sec^2 t \tan t} dt = \frac{1}{16}\int \cos t dt = \frac{\sin t}{16} + C,$$

Now $t = \arccos(4/x)$, so $\sin t = \sqrt{1-\cos^2 t}$, and we have

$$I = \frac{\sqrt{1-16/x^2}}{16} + C = \frac{\sqrt{x^2-16}}{16x} + C.$$

(3) Evaluate the integral

$$\int \frac{dx}{\sqrt{1+x}+\sqrt{x}}.$$

(You don't need to change variables, you can do algebraic manipulation using the identity $(a+b)(a-b) = a^2 - b^2$).

Following the suggestion, we note that

$$\frac{1}{\sqrt{1+x}+\sqrt{x}} = \frac{1}{\sqrt{1+x}+\sqrt{x}} \frac{\sqrt{1+x}-\sqrt{x}}{\sqrt{1+x}-\sqrt{x}} = \sqrt{1+x} - \sqrt{x}.$$

Therefore

$$I = \frac{2}{3}\left((1+x)^{3/2} - x^{3/2}\right) + C.$$

(4) Evaluate the improper integral

$$\int_0^{100} \sqrt{t} \ln t dt.$$

Integrate by parts

$$\int_0^{100} \sqrt{t} \ln t dt = \frac{2}{3} t^{3/2} \ln t \Big|_0^{100} - \frac{2}{3}\int_0^{100} t^{1/2} dt = \frac{2000}{3}\ln 100 - \frac{4000}{9} = \frac{4000}{3}\left(\ln 10 - \frac{1}{3}\right).$$

Note that we have used L'Hospital's rule to compute

$$\lim_{t\to 0} t^{3/2}\ln t = \lim_{t\to 0}\frac{\ln t}{t^{-3/2}} = \lim_{t\to 0}\frac{\frac{1}{t}}{-\frac{3}{2}t^{-5/2}} = 0.$$

Alternatively, you can substitute $u = \ln\sqrt{t} = \frac{1}{2}\ln t$ and then $t = e^{2u}$, so $dt = 2e^{2u}du$

$$\int_0^{100}\sqrt{t}\ln t dt = \int_{-\infty}^{\ln 10} e^u 2u 2e^{2u} du = 4\int_{-\infty}^{\ln 10} e^{3u} u du = \frac{4}{3}e^{3u}(u-\frac{1}{3})\Big|_{-\infty}^{\ln 10} = \frac{4000}{3}(\ln 10 - \frac{1}{3}).$$

(5) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis:

$$y = 2,\ y = x^2;\ \text{about the } y \text{ axis.}$$

We'll use y as the independent variable, and so

$$V = \pi\int_0^2 x^2(y)dy = \pi\int_0^2 ydy = 2\pi$$