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HONORS CALC II F 08, QUIZ #9

- (1) For each of the following series determine whether it converges absolutely, converges conditionally or diverges.

$$\sum_{n=2}^{\infty} \left(\frac{2n}{n+1} \right)^{n^2}$$

$$(b) \sum_{n=1}^{\infty} \sqrt{n} \ln \left(1 + \frac{1}{n^2} \right)$$

(a) Use root test to and observe that $\sqrt[n]{a_n} = 2^n \left(\frac{n}{n+1} \right)^n \rightarrow \frac{\infty}{e} = \infty$, so the series diverges. The hint provided was somehow misleading (or at least not helpful). Maybe because of it, some of you solved different problem (what I originally intended to give you), and considered $a_n = 2^n \left(\frac{2n}{n+1} \right)^{n^2}$. With this choice $\sqrt[n]{a_n} = 2 \left(\frac{n}{n+1} \right)^n \rightarrow 2/e < 1$, and so the series converges absolutely. I accepted both answers.

(b) Use L'Hospital to show that $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$. Hence letting $x = \frac{1}{n^2}$ and choosing $b_n = \sqrt{n} \frac{1}{n^2} = n^{-3/2}$ we have $a_n/b_n \rightarrow 1$. Thus both $\sum a_n$ and $\sum b_n$ converge or diverge simultaneously. The latter is a p -series with $p > 1$, and therefore it converges. Hence $\sum a_n$ converges.

- (2) Find a power series representation about $a = 1$ for the function $f(x) = \frac{2}{3-x}$ and determine its radius of convergence and its interval of convergence.

We wish to express f as a series of powers of $x - a = x - 1$. We first arrange the function accordingly:

$$\frac{2}{3-x} = \frac{2}{2-(x-1)} = \frac{1}{1-(x-1)/2} = \sum_{n=0}^{\infty} 2^{-n} (x-1)^n.$$

We have $c_n = 2^{-n}$ and so $R = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n]{c_n}} = 2$. The series converges absolutely on $(1-2, 1+2) = (-1, 3)$. We show that this is also the interval of convergence. Indeed, at the endpoints $x = -1, 3$ we have $|c_n(x-1)^n| = 1$ and so the general term does not converge to 0 and the series diverges.