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PDE FOR APPS F 08, QUIZ #8

(1) Suppose we wish to solve the problem

$$\begin{cases} 4u_{xx}(x,t) = u_t(x,t) & 0 < x < 1,\ t > 0 \\ u(0,t) = 0,\ u_x(1,t) + u(1,t) = 0 & t \geq 0 \\ u(x,0) = f(x) & 0 \leq x \leq 1. \end{cases}$$

where $f(x)$ is some given function. We have seen that we can write the solution $u(x,t) = \sum_{k=0}^{\infty} c_k u_k(x,t)$, where c_k's are constants determined by f and $u_k(x,t)$ are obtained by separation of variables. That is, $u_k(x,t) = X(x)T(t)$ for some functions X, T (which depend on k).
Write the equations $X(x)$ and $T(t)$ should satisfy, including boundary/inital conditions (if any apply). You are not required to solve the equations.

We assume $u_k(x,t) = X(x)T(t)$. The differential equation implies $4X''(x)T(t) = X(x)T'(t)$. Separating variables we obtain $\frac{X''(x)}{X(x)} = \frac{T'(t)}{4T(t)}$. Left-hand side is a function of x, while right-hand side is a function of t. Therefore each has to be equal to some constant, say $-\lambda$. Apply the homogeneous boundary conditions. Since $0 = u(0,t) = X(0)T(t)$, we conclude that $X(0) = 0$, and since $0 = u_x(1,t) + u(1,t) = X'(1)T(t) + X(1)T(t)$, we conclude that $X'(1) + X(1) = 0$. The non-homogeneous condition $u(x,0) = f(x)$ is not applied at this stage and will be used only to determine the coefficients c_k. Summarizing:

$$\begin{cases} X''(x) + \lambda X(x) = 0 \quad X(0) = X'(1) + X(1) = 0 \\ T'(t) + 4\lambda T(t) = 0 \end{cases}$$