Truth and paradox: a map of major divide

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In my judgment, it would be quite wrong and dangerous from the standpoint of scientific progress to depreciate the importance of this [Liar] and other antinomies, and to treat them as jokes or sophistries.

– Alfred Tarski [15]

One way of thinking about the significance of the truth-theoretic paradoxes turns on the background conception of truth. (This can in turn be used to evaluate solutions to the paradox.) There are undoubtedly many different ways to carve things up, but I will use one that seems to be both historically well-grounded (or, at least, not historically aberrant) and conceptually useful. In particular, I shall use a two-fold distinction: semantic truth and transparent truth.¹

Following Field [3], McGee explicitly notes the (or, at least, a) divide to which I’m gesturing:

We discern two separate aims one might have in developing a theory of truth. One is to explain the processes by which language is linked with the world. The other is to describe the language in sufficient detail to get the T-sentences. There is no good reason to suppose that a theory that fulfills one of these aims will fulfill the other. [13, p. 622]

I would change this slightly to make the mapping point I’m advancing: instead of ‘getting the T-sentences’, I’d say that the other discernible aim is to get transparency, rather than merely T-sentences. (See below.) But otherwise, I think that McGee and I agree on an important divergence of motivations here. (Still, what follows is my own thinking. If McGee already said what I’m saying, then I agree with McGee! Otherwise, this is a suggestion towards useful classification.)

¹This is used in previous work with Michael Glanzberg, although I haven’t thought hard about whether what I say here differs from what we say there. See [2].

²Tarski himself [15] also notes different conceptions, and encourages working them out (as a sort of ‘truth pluralism’); however, it is not at all clear that Tarski is talking about doing all of this within a single language, unlike McGee (and my own interests).
1 The semantic conception

Here, the importance of truth is its explanatory work in semantics: we aim to give truth conditions, the conditions under which sentences are true.\(^3\) Truth, along these lines, is some important (semantic) relation between sentences and the world (or, perhaps, bits of the world, situations, whatever).

On this (the target ‘semantic’) conception of truth and its role, truth is the (or, at least, one of the) chief language-to-world relations invoked in doing semantics.\(^4\) We cannot provide the meaning of (declarative) sentences without talking about truth, in particular about the conditions in the world (concrete, abstract, whatever) under which sentences are true (stand in this important relation). So (see initial McGee reading), if there are some (meaningful, declarative) sentences for which we cannot give truth conditions, our science of semantics is problematic; and if the crucial notion of truth cannot itself enjoy proper truth-conditional treatments (e.g., giving truth conditions for all declarative sentences that use ‘true’), then this is problematic. (Again, see McGee.)

For a sense of the target semantic conception, see Dowty et al. ‘intro’; and see Tarski PPR ‘Semantic Conception’;\(^5\) and see other discussions in recommended readings like Soames, Gupta–Belnap, and perhaps particularly Barwise & Etchemendy.

1.1 Significance of truth-theoretic paradoxes

In short, the significance of truth-theoretic paradoxes is that they challenge our theories (understandings) of meaning, and hence (as McGee says) our theories of communication, of human interaction, and so on. [See specifically p. 628 3rd para, and also re: naturalism in 4th para of the ‘Truth and Paradox’ piece.]

Presumably, the adequacy of one’s account of truth needs to be measured by one’s account of semantics and, in particular, the role that truth is supposed to play there. (One could do this the other way around: find out what ‘semantic truth’ is, and let that determine semantics. But it seems to me that the circle often goes the other way around.)

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\(^3\)Perhaps this is to be generalized to satisfaction conditions that apply to all sentences, not simply (as I assume throughout) ‘declarative sentences’. But I ignore this throughout.

\(^4\)NB: for all I’ve said, the property of truth itself can be a base, primitive property that enjoys no ‘analysis’. (E.g., the world relates to THE TRUE. And there’s stuff to say about the world-side of this, but nothing to say about the truth-property bit.) Historically, I think, folks have thought that there’s something to say about the truth-side.

\(^5\)Note, though, that Tarski – perhaps unfortunately – talked of formulating ‘the conditions under which a sentence of the language can be asserted’ (see his §6), but it may be that he was trying to avoid using ‘true’ because his task was to define it?? (Or maybe this just shows that he was not doing truth-conditional semantics??) Moreover, Tarski himself was apparently chiefly concerned not with natural languages but with formal language – or, perhaps, formalized fragments of natural languages (and, so, perhaps not exactly unlike Montague’s project). . . . Let me note that, in general, I find Tarski’s essay somewhat confusing (e.g., the assertability stuff in §6 may be at odds with remarks in §18, etc.). I’m not concerned here with Tarski exegesis.
1.2 The role of truth-in-a-model conditions

The importance of formal semantics (in effect, model theories for various formal languages) lies in its modeling work: we give truth-in-a-model conditions as a model of truth conditions. As such, if one can show that one’s guiding (semantic) notion of truth is not modeled by one’s proffered truth in a model notion, then one’s proffered account has a prima facie difficulty. [As we’ll see, so-called ‘revenge problems’ often arise along this front.]

Check out Hodges 1986 Proc. Arist. Soc. ‘Truth in a structure’ paper. (It may or may not be of use here.)

2 Transparent conception

Here, the importance truth is not that it is an important, explanatory notion; its importance is only as a see-through (transparent) device of generalization. As Tarski [15] himself noted—and later Quine [14] and, forcefully, Leeds [10]—we need some device that allows us to generalize over sentences in nominal position: we want to say something that implies (say) everything in So-in-so’s theory without explicitly asserting everything in the theory. We do this by saying that everything in So-in-so’s theory is true. More plainly: we may have need to generalize over all sentences of the form \( A \lor \neg A \), and we may want to say something that implies all such sentences; but there are too many of them and too little time to explicitly assert each such sentence; and so we instead simply say that all such sentences are true.

The idea, in short, is that we have a practical (versus, e.g., semantic-theoretical) need for a transparent device of generalization. And this, on the target transparent conception, is where the (target) notion of truth comes into play. We don’t introduce ‘true’ to pick out some important (semantic or other sort of) property in the world; we bring it in only to serve the given practical (expressive) need. At the very least, the target transparent device \( Tr(x) \) is one in which \( Tr(\langle A \rangle) \) and \( A \) are equivalent (in some suitably strong sense); we need the predicate to be redundant in principle. (Here, the corner quotes are used for an appropriate naming device—quotation, Gödel codes, whatever.)

·· Parenthetical remark. Exactly what sense of equivalent is used is important, and can distinguish different sorts of ‘transparent theorists’. Actually, my own usage of the term ‘transparent’ is fairly restricted. A transparent truth theorist, on my usage in [1], maintains that the given device is genuinely see-through in the following sense: for all sentences \( A \) of the given language, \( Tr(\langle A \rangle) \) and \( A \) are intersubstitutable in all (non-opaque) contexts. This can be put: \( B \) and \( \hat{B} \) are equivalent, where \( \hat{B} \) is the result of substituting any occurrence of \( A \) in \( B \) with \( Tr(\langle A \rangle) \). ....I’ll figure out exact terminology later. End parenthetical. ··

For present purposes, what is important is that the transparent conception (however exactly formulated) is very different from the semantic conception: the latter concerns an important semantic property, one essential to semantic theory (particularly qua explanatory property essentially involved in truth conditions):
the former concerns not an important semantic property, etc., but rather only a practical, see-through device brought into the language to overcome in-practice (but not in-principle) shortcomings (finitude of our time, minds, whatever).

2.1 Significance of truth-theoretic paradoxes

Given that the target truth predicate is in place for practical (versus theoretical-explanatory) reasons, the significance of the truth-theoretic paradoxes concerns the effects it has on the (practical) role of the device. In short: how, if at all, do we enjoy a see-through device in our language without it leading (via its paradoxes) to absurdity?

Presumably, the adequacy of one’s account of see-through truth is to be measured not only in its achieving transparency (see-through-ness over the whole language), but the ‘naturalness’ in which this is achieved. (Presumably, this is all tied up with holistic factors that Quine emphasized, etc.)

2.2 The role of truth-in-a-model conditions

It is not clear what role truth-in-a-model conditions play for the transparent conception. If truth is not an explanatory notion, then it is not obvious why truth-in-a-model should model truth (or what this would even amount to here).

Of course, model-theoretic-semantics might be given as a heuristic for thinking about the logic of the truth theory; however, it is far from clear why—or in what way—it would be taken as reflecting ‘reality’ in the way that traditional truth-in-a-model conditions have been taken to model ‘real truth conditions’ etc.

Unlike in the ‘semantic conception’, where truth-in-a-model is taken to model truth, it remains an open—and very important—question as to how, if at all, truth-in-a-model might play more than a merely heuristic role with respect to truth theories in the ‘transparent’ conception.

3 Truth theories along the given divide

We can now divide major truth theories and their approach to paradox along the given divide. (NB: there are many more theories than those waved at here, but the listed theories are the major ones.) There are different ways to classify the given theories (e.g., classical-logic versus non-classical, or fixed-point versus hierarchical versus revision, and other such classifications). These other ways of classifying can be useful and illuminating. (I should also note that the different ways of classifying take different stands on very fundamental, difficult issues—e.g., status of logic, etc.) The classification that I’m suggesting here is a supplemental one that I hope will be useful.

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6Of course, the see-through device plays a role in explanations—namely, its generalizing role etc—but it doesn’t serve as an explanatory notion itself in the way that the target ‘semantic’ notion of truth is supposed to do.
1. Semantic conception

(a) Tarskian hierarchy of languages: here, we can only do semantics (provide truth conditions) for languages ‘below’ us. See [13] and [15].

(b) Contextualist (hierarchy of contexts): here, our semantic notion of truth is contextually sensitive; and—as in Tarski’s limitations—we can never stand in a context that provides what we might think of as a ‘complete semantics’ for the whole language. (This may be misleading; we can think harder when we see the paper(s).) See [6].

(c) Revision theories: here, our semantic notion of truth is a revision-theoretic one; it is governed by ‘rules of revision’. (Here, we face perhaps not limitations about what we can say about truth, but apparently other notions—e.g., stability or the like—that appear, on this approach, to be as fundamental to semantics as truth.) See [7].

(d) Certain ‘Determinacy’ theories (e.g., McGee’s) along classical-logic lines (e.g., closed-off Kripke) See [12].

2. Transparent conception

(a) Paraconsistent theories: here, our device is see-through in virtue of a paraconsistent logic. (Issues arise re: a suitable conditional and other ‘semantic’ notions like validity, etc.) See [1, Ch. 1–3].

(b) Paracomplete theories: here, our device is see-through in virtue of a paracomplete logic. (Issues arise re: a suitable conditional and other ‘semantic’ notions like validity, etc.) See [9], [4, 5] and [1, Ch. 4].

Note very well: these categories are not mutually exclusive! (E.g., there are paraconsistent theories that are guided by a sort of semantic conception—e.g., Priest’s pioneering dialethic theory is driven by a Dummettian teleological theory. Similarly, McGee’s paracomplete theory could be easily placed, in one way, on the transparent conception—except for his ‘semantic’ determinate truth notion.) But the divide is useful for thinking about motivation and evaluation of the given approaches.

References


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7 This is more for Tarski Star (a sort of idealized deviation from the historic Tarski whose work applied to formal languages only).

8 This is reserved for discussion only after the transparency views.


