Logic, Paradox, and Truth

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AIMS

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- Logical Theory: what sorts of things does a logical theory tell us?
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- Concluding reflections...
**LOGICAL TRUTH AND RELATIONS**

Logic concerns (logical) implication among statements. Example: 1. All birds have wings. 2. Kiwis are birds. 3. Kiwis have wings. The relation between (1) and (2) together and (3) is logical implication: there's no way for (1) and (2) to be true without (3) being true – and this due to logical words. Some key logical words are 'All', 'there exists', 'is' (exemplifies), 'not', and more.
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Logical Truth (and General Laws)

Given a stock of logical words, we also get logical truths: true statements that are true merely in virtue of the logical words involved.

Examples (of generalizations of logical truths):

▶ LNC: No statement is both true and not true.
▶ LEM: Every statement is either true or not true.
▶ Identity: If a statement is true then it is true.
▶ Double negation: If a statement is true, then it is not not true (and vice versa).
▶ . . . and there are many others.
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SUMMARY: LOGIC

Logical implication is an airtight truth-related connection between statements that holds merely in virtue of logical words.

Logical truths are statements that are true merely in virtue of logical words.

Among currently recognized logical words are 'not', 'or', 'and', 'if...then', 'all', 'there exists', and others.

In general: logic is about certain properties and relations that hold among statements in virtue of basic 'logical words'.
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LOGICAL THEORIES: LAWS AND RULES
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**Logical Theories: laws and rules**

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〈NB: The form that such laws/rules take in contemporary logical theory are often rather mathematical and symbol-ridden. We shall avoid all that here.〉
**Challenge to LNC?**
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Law of Non-Contradiction (LNC):
No statement is both true and not true.
CHALLENGE TO LNC?

Law of Non-Contradiction (LNC):
No statement is both true and not true.

✓ The ticked statement is not true.
**Challenge to LEM?**

Law of Excluded Middle (LEM): Every statement is either true or not true. ✓✓

The double-ticked statement is true.
Challenge to LEM?

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CHALLENGE TO EXISTENTIAL GENERALIZATION?
Challenge to Existential Generalization?

Rule of Existential Generalization (EG):
From So-n-so is $G$ infer There exists an object that is $G$. 
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From *So-n-so is G* infer *There exists an object that is G.*

*Harry Potter is fictional.*
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Even the most obvious logical laws and rules have apparent challenges.

We've looked at apparent challenges to two laws (LNC and LEM) and one rule (EG).

How (if at all) we should adjust our logical theory is not easy – and an ongoing issue.

Ultimately, logical theory is much like any other theory: it is subject to the pressures of reality – including the weird, limiting pockets thereof.

My own view is that there is exactly one true logical theory (even if many logics), but the quest for it remains open.
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Summary and closing remark

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Like the quest for the one true logical theory, *discussion period* is now open...