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Comment on Gila Sher's "Is Logic in the Mind or in the World?"

The central issue of Sher's paper is the claim that logic is, at least to a significant extent, substantially grounded in the world or in reality (where those terms are used interchangeably).

Schematically, Sher represents her proposal like this:

(Level of Logic) Γ *logically implies* σ

$\Uparrow\Downarrow$

(Level of Truth) $T(\Gamma) \Rightarrow\Rightarrow\Rightarrow T(\sigma)$

$\Uparrow\Downarrow$

(Level of Reality) $\mathfrak{S}_\Gamma \rightarrow\rightarrow\rightarrow \mathfrak{S}_\sigma$

The $\mathfrak{S}$'s stand for features of reality, situations[1] in the world, whose indices indicate that the sentences Γ and the sentence σ correspond to just these situations. '$\rightarrow\rightarrow\rightarrow$' stands for a (worldly) strong necessitation relation, 'T' asserts the truth of what occurs in its argument place, and '$\Rightarrow\Rightarrow\Rightarrow$' indicates that the truth on the left-hand side guarantees in a strong sense the truth on the right-hand side.

Sher describes in detail how the $\Uparrow\Downarrow$-dependencies are supposed to ground the logical implication of σ by Γ in reality, and there is no point in rehearsing the picture in full detail again, not least for the reason that it is not overly likely that I would do a better job at it than Sher does herself.

[1] I allow myself the use of 'situation' here and in the following — I do not intend a technical notion as, for instance, in situation semantics.

I am sympathetic towards this project: its success would reconcile the applicability of logic with objectivity and normative force, which would be quite a feat. My contention is, however, that the hope for such a grounding is too optimistic. In what follows I will voice two concerns, one about each of the directions of the ⇑⇓-dependency, and for this purpose I will retrace some of the steps that lead to the diagram above.

1. ⇓

The downward direction indicates, and is obtained by observing that, if the necessitation relation does not obtain on the level of reality, the corresponding relations on the levels of truth and logic do not obtain either:

(Level of Logic) **NOT:** (Γ *logically implies* σ)

⇑

(Level of Truth) **NOT:** ($T(\Gamma)$ ⇨⇨⇨ $T(\sigma)$)

⇑

(Level of Reality) **NOT:** ($\mathfrak{S}_\Gamma$ →→→ $\mathfrak{S}_\sigma$)

As a concrete example, Sher offers:

(Level of Logic) **NOT:** (‘$\exists x(Px \wedge Qx)$’ *logically implies* ‘$\exists x\, Px$’)

⇑

(Level of Truth) **NOT:** (T(‘$\exists x(Px \wedge Qx)$’) ⇨⇨⇨ T(‘$\exists x\, Px$’))

⇑

(Level of Reality) **NOT:** (**nonempty($\mathfrak{P} \cap \mathfrak{Q}$) →→→ nonempty($\mathfrak{P}$)**)

(Sher carefully distinguished various relations indicated by different styles of arrow triples and turnstiles so that the relations on the three levels correspond. Since nothing much hangs on these distinctions for *my* discussion, I allow myself to be somewhat sloppier here.)

My concern is rather simple: I do not think that the appeal of this picture has anything to do with a grounding of the failure of logical implication in reality, but rather with the more general appeal that counter-models for logical implication have. If logical implication at least means truth-preservation, then showing that the premises of an argument can be true while the conclusion is false demonstrates that there is no logical implication between the premises and the conclusion. There is no requirement to find an *actual* situation in the world that provides a counterexample — and for well-known reasons that had better not be the case anyway. Thus, while finding that in fact **NOT:** ($\mathfrak{nonempty}(\mathfrak{P}\cap\mathfrak{Q})$ $\rightarrow\rightarrow\rightarrow$ $\mathfrak{nonempty}(\mathfrak{P})$) does show that **NOT:** ('$\exists x(Px \wedge Qx)$' *logically implies* '$\exists x\ Px$'), any two merely possible situation that stand in the '$\rightarrow\rightarrow\rightarrow$'-relation would have done the same trick. No evidence for a grounding in reality is available in this way.

This is too quick as a rebuttal, however. Sher stresses the *inherent* connection that logical implication has to have with the '$\rightarrow\rightarrow\rightarrow$'-relation, so the situations themselves may not be meant to do the work here. Perhaps, the situations need not actually obtain, so the grounding does not have its source in the observation that, if we find that $\mathfrak{nonempty}(\mathfrak{P}\cap\mathfrak{Q})$ but not that $\mathfrak{nonempty}(\mathfrak{P})$ we have found a counterexample to " '$\exists x(Px \wedge Qx)$' *logically implies* '$\exists x\ Px$' ". How, then, is the grounding achieved? The intimate connection between $\rightarrow\rightarrow\rightarrow$ and logical implication is meant to fall out of the above observations (here: for the $\Downarrow$-direction) and hence cannot be presupposed. My more general discussion of the $\Uparrow$-direction of the grounding will shed further light on this. Let me, however, re-emphasize that the initial appeal that the $\Downarrow$-direction of the picture has, has nothing to do with a grounding of logical consequence — while admitting that it is not meant to work in this way either.

2. $\Uparrow$

The inference from '$\exists x(Px \wedge Qx)$' to '$\exists x\, Px$' is, in fact, a case of logical implication of course. Logical implication is not merely constrained by the world in the way suggested above, but also "empowered" by it, as Sher puts it. That '$\exists x(Px \wedge Qx)$' logically implies '$\exists x\, Px$' is, according to Sher, grounded in the fact that the $\rightarrow\rightarrow\rightarrow$-relation obtains between the worldly facts that correspond to the sentences. Diagrammatically:

(Level of Logic) Γ *logically implies* σ

$\Uparrow$

(Level of Truth) $T(\Gamma) \Rightarrow\Rightarrow\Rightarrow T(\sigma)$

$\Uparrow$

(Level of Reality) $\mathfrak{S}_{\Gamma} \rightarrow\rightarrow\rightarrow \mathfrak{S}_{\sigma}$

And more concretely:

(Level of Logic) '$\exists x(Px \wedge Qx)$' *logically implies* '$\exists x(Px)$'

$\Uparrow$

(Level of Truth) $T(\text{'}\exists x(Px \wedge Qx)\text{'}) \Rightarrow\Rightarrow\Rightarrow T(\text{'}\exists x(Px)\text{'})$

$\Uparrow$

(Level of Reality) $\mathfrak{nonempty}(\mathfrak{P}\cap\mathfrak{Q}) \rightarrow\rightarrow\rightarrow \mathfrak{nonempty}(\mathfrak{P})$

Let me first emphasize that I do not take Sher to be claiming that $\rightarrow\rightarrow\rightarrow$ in some sense just *is* logical consequence, therefore a basic *part* of the world, and so *trivially* (because we use 'world' and 'reality' interchangeably) grounded in reality. Sher's detailed discussion of the properties of '$\rightarrow\rightarrow\rightarrow$' prevents such an interpretation; further, since the grounding relation is to be irreflexive (p. 8), identity is disqualified from being a grounding relation. (Such a proposal would, I take it, also be a lot less interesting.)

Let me, parenthetically, as it were, remark however that some passages in Sher's paper might tempt the hasty reader to interpret the proposal in this way, at least in parts. Prominent amongst such passages is the example of Frege's logic of the *Grundgesetze*, which failed famously and dramatically. Sher diagnosis of the failure is that "Frege's logic is committed to the *existence* of an object — a class — that does not, and cannot, exist" (p. 8, emphasis in the original). According to the Frege of the first volume of *Grundgesetze*, all of his Basic Laws are logical laws, and the extensions (or classes) mentioned in Law V as *logical* objects. In fact, even after Russell's devastating discovery Frege did not (at least not immediately) give up on the logicality of extensions — although he probably wished he had not written in the preface to the first volume that

> "a dispute can arise only concerning my Basic Law of value-ranges (V), which perhaps has not yet been explicitly formulated by logicians although one thinks in accordance with it if, for example, one speaks of extensions of concepts. I take it to be purely logical." (Frege, *Grundgesetze,* vol. I, p. VII)
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> "I would only count it as a refutation if someone could indeed demonstrate that a better, more enduring building could be founded on different basic convictions, or if someone proved to me that my basic principles lead to manifestly false conclusions. But no one will succeed in this." (Frege, *Grundgesetze,* vol. I, p. XXVI)

Sher indeed speaks of the whole system of *Grundgesetze* as "Frege's logic", just in the way Frege himself thought of it, and indeed, as we now know, the system without Basic Law V is consistent and essentially just what we now call (classical) second-order logic. Thus, if classes are *logical* objects in this way, then the Frege case cannot show that logic is grounded in reality. The dilemma I see is this: If classes are logical objects and in the world, then logic would have to be part of the world, and hence cannot be grounded in it. If, on the other hand, classes are *not* logical, then the example does not show that logic is grounded in reality, since it would be an extra-logical feature of the system that has been refuted (assuming Sher's analysis of the problem with Frege's system is correct).

But this is a mere quarrel about a mere example that is in no way central to Sher's arguments. My real concern lies elsewhere: in order to ground logical consequence in the worldly →→→-relation in a sufficiently strict and substantial way, →→→ must fulfill some strong requirements. The properties Sher suggests are "necessity, formality, relative apriority, topic neutrality, strong normative force, great generality, and certainty" (p. 12). I do not envy Sher for the task to come up with characterizations of, for instance, necessity and generality, that are needed in order to complete the account, if avoiding the charge of circularity should mean spelling out these notions without using logical consequence in any substantial way. But the *prima facie* most baffling demand is that of *formality* for the worldly →→→-relation. This is what I want to take issue with in the following.

Formality is most easily thought of as a feature of a language (*formal* language) or logic (*formal* logic). The multitude of attempts to spell out what the formality of logic amounts to (cf. MacFarlane) demonstrates that even here it is far from clear or uncontroversial how the notion is to be defined or otherwise made precise. Sher however has an intriguing answer to the question what formality might mean when we are trying to characterize a relation in reality with it. It is about the structure of reality, and the relevant structural features are characterized by, what else, the science of structures, i.e. mathematics. Initially, the instruments will be *basic* mathematical notions, like meet, join, identity or finitude. (Indeed, one may note the meet in the formulation of the example above. Negation and non-emptiness, presumably, belong in the basic category too — or we may get these, further up, from complement and identity.)

Sher anticipates the flat-footed criticism that mathematics is not to be had without logic, and rightly notes that, since her project is not foundational in the strict sense, the priority of one over the other does not need to be assumed: both disciplines are to be developed in tandem.

Equally flat-footed may be the complaint that what we find at the bottom of Sher's diagrams appear to be (maybe mathematical) representations of reality already, rather than the world itself. I take it that we want to allow Sher to speak about the world when she presents her account. How else, but through language or other signs, can one convey one's theory? So I hope that my at first blush perhaps similar sounding objection below will not be confused with this quixotic complaint.

A host of philosophers have argued for theses that seem to undermine Sher's project. If at least some of these are to be taken serious, then Sher may be faced with problems for the grounding claim. Think of Quine's question whether "*gavagai*" denotes rabbits or rabbit-stages, or whether it may be construed in a not object-involving fashion by way of an environmental predicate: it's rabbiting. For Carnap, all comes down to a question of the choice of what he calls "languages" ("systems" may be more instructive); a Putnamian might be concerned with differences in conceptual schemes; and there are Goodman's many world-versions, including *green*-worlds and *grue*-worlds, which may contain mutually incompatible nomic connections. It may be objected here that these positions all point to a relativity pertaining to the second of Sher's levels, the level of truth, since all of these positions may be construed as concerning languages or representations, rather then the world. Maybe the grounding relation can wiggle its way through this mess to the comparatively more solid lowest level, the level of reality.

Mathematics is the tool that Sher suggests we use to characterize the structure which is to ground logical consequence. Mathematics is to operate on the level of reality, not the level of truth (or language). (While one may doubt whether mathematics is indeed applicable without prior decisions regarding the representation of reality — which may well bring Carnap's, Quine's, Goodman's, and Putman's concern back into the picture — I will here not presuppose this.)

Now, one may point to recent discussions in the blooming new field of meta-ontology that provide a plethora of fresh examples which, at least at the face of it, may present problems to the notion that logical consequence is grounded in reality. Eli Hirsch, for instance, contends that the notion of an object itself can be carved out in many mutual contradictory ways using different conceptions of Mereology. The proponent of so-called *unrestricted mereological composition* (think Goodman or David Lewis) holds that the mereological sum of *any* objects composes an object, no matter how scattered; van Inwagen suggests that the only objects that exist except of simples (objects that do not have any proper parts) are organisms (people, fungi, animals, plants, …, but not tables or chairs); others (including Hirsch) favour a "common-sense" ontology that admits of any medium-sized dry good as an object, but not scattered objects. Hirsch's claim is that all these different ways of carving up reality are expressively adequate, inter-translatable[2] and empirically equivalent. "Object", however, appears to be a fairly basic ontological category and, moreover, one that is badly needed: think of Tarski's permutation test for logical constants (which is also championed by Sher) which requires one-one replacements of *objects*.

If we are by Hirsch's contention free to pick whatever mereological structure we like (within reason) to determine our notion of an object, then this part of reality at least does not appear to supply enough structure of its own, or so one may argue. Mereological notions, like sum, should be basic enough, however, to qualify as basic structure exhibiting tools: sum is, essentially, join for objects. If indeed we possess this much freedom regarding this selection, then this casts doubt on whether something can be found in reality at all to ground, say, interferences pertaining to the existential quantifier.

Even if we are inclined to reject such speculative metaphysics, there are still

[2] As presented elsewhere, I have strong doubts that this is the case, and Hirsch has yet to propose a systematic translations scheme. The example may nonetheless raise worries.

more pressing problems looming. Consider, for example, constructivist mathematics. Mathematics is meant to reveal the structure of reality and this enable us to ground logic in a worldly strong-necessitation relation. Suppose the constructivists' claim that constructivist mathematics can be developed to a point where it suffices for all (present and future) scientific purposes. We would, then, be in the position where we could on empirical grounds no longer favour classical over constructivist mathematics as revealing the structure of reality. Presumably, intuitionist logic will be the logic that turns out to be grounded in reality, if the structure of reality is characterized by constructivist mathematics. Classical mathematics will, presumably, allow us to ground classical logic in reality. Thus both would be grounded in reality, yet they disagree.

For a more concrete example, consider the case against Euclidian geometry that the advent of Einstein's Relativity provided. It was generally held that the structure of space was characterized by Euclidian geometry when Newtonian physics was believed to be correct. Special Relativity is usually thought to have demonstrated that this conception of space was wrong, that the structure of spacetime has to be indeed non-Euclidian.

Consider Poincaré's and Reichenbach's arguments, however, that Special Relativity shows no such thing. In a nutshell, the claim is that a non-Euclidian geometry allows us to have a simple and elegant physical theory, but that for the price of an awkward and cumbersome physical theory one could stick to Euclidian geometry instead. The resulting theory, despite its ugliness, is empirically equivalent to the beautiful non-Euclidian theory (*pace* Malament).

The more extreme moral that some may wish to draw from this is that mathematics does not help *discover* the structure of reality, but rather *imposes* structure on reality, and that we have considerable freedom in the choice of structures that we want to give the world. A more modest moral is that the world does not determine in sufficient detail which mathematical theory to pick to represent it; it does not force our choice to the extent that we might have hoped for: uniquely. The worry thus is that the only grounding that is to be had is grounding in *representation* after all — a grounding in conceptual(ized) reality, rather than reality *per se*.

My question to Sher is thus: given the choice in structure that these examples suggest we have, can logical consequence be *grounded* in a worldly strong-necessitation relation, whose crucial feature is *formality* which, in turn, is spelled out in terms of *the* structure of reality?