Logical Consequence: From Logical Terms to Semantic Constraints
Abstract for the Models in Formal Semantics and Pragmatics Workshop

In the paper I propose a new framework for extensional logic, where the explication of the notion of logical consequence is the primary aim. The framework may also be applicable in the study of natural language, especially in illuminating various semantic relations between expressions.

In the paper I discuss a prevailing view by which logical terms determine the forms of sentences and therefore the logical validity of arguments. This view is common to those who hold that there is a principled distinction between logical and nonlogical terms and those holding relativistic accounts. I adopt the Tarskian tradition by which logical validity is determined by form, but reject the centrality of logical terms. I propose an alternative framework for logic where logical terms no longer play a distinctive role. This account employs a new notion of semantic constraints. The paper includes some preliminary definitions and results in the new framework.

A semantic constraint for L is a sentence in the metalanguage that somehow constrains or limits the admissible models for L (and can be viewed as a meaning rule). Logical terms (or more precisely, rules defining logical terms) are merely a special case of semantic constraints, while all the semantic constraints in a system are involved in determining logical consequence.

Take, for instance, the terms allRed and allGreen.\footnote{Consider allRed allGreen as a short for red all over and green all over.} These are paradigmatic cases of nonlogical terms in mainstream logic. There are good reasons for not fixing the extensions of color-terms completely. But we could fix their mutual dependencies, and have a rule in our system that says that the intersection of their extensions is empty in all models. A rule like this, I contend, is not essentially different from a rule fixing the interpretation of a logical term. In both cases, there is a rule that consists in restricting admissible models. We may say that whereas the interpretations for logical terms are completely fixed, other terms may be partly fixed.

Let L be a language including a set of terms (the primitive expressions of the language) and phrases (strings of terms, possibly including auxiliary devices such as parentheses). Let a model for L be a pair \((D,I)\) where \(D\) is a nonempty set (the domain), and \(I\) an interpretation function, assigning extensions to phrases in L. As we deal here with models set up on a classical set-theoretic foundation, the metalanguage used in all the examples includes, but is not confined to, the language of set theory. Semantic constraints include implicit universal quantification over models (domains and interpretation functions).
The semantic constraint mentioned informally previously can be formulated thus:

\[ I(\text{allRed}) \cap I(\text{allGreen}) = \emptyset \]

The semantic constraint for standard conjunction will be:

\[ I(\phi \land \psi) = T \iff I(\phi) = T \text{ and } I(\psi) = T \]

Semantic constraints are similar to meaning postulates in their function. However, they are formulated in the metalanguage, and thus have more expressive power. Moreover, they allow more flexibility, and need not assume a distinction between logical and nonlogical terms.

In the paper I show how standard propositional and first order logic can be formulated as systems of constraints. These systems can then be modified by either eliminating some of the constraints or adding non-standard ones.

The framework, despite its simplicity, provides a wealth of formal definitions and results concerning semantic properties and relations between expressions. I give some examples below.

Let \( \Delta \) be a set of semantic constraints. A \( \Delta \)-model is an admissible model by \( \Delta \), i.e. a model abiding by the constraints in \( \Delta \).

**Definition 1** (Validity) An argument \( \langle \Gamma, \phi \rangle \) (where \( \Gamma \) is a set of sentences in \( L \) and \( \phi \) is a sentence in \( L \)) is valid (w.r.t. \( \Delta \)) if for any \( \Delta \)-model (an admissible model for \( \Delta \), satisfying those constraints), if all the sentences in \( \Gamma \) are true, then \( \phi \) is true as well.

**Definition 2** (Dependency) A set of phrases \( A \) depends on the set of phrases \( B \) (w.r.t. \( \Delta \)) if there are \( \Delta \)-models \( M = \langle D, I \rangle \) and \( M' = \langle D, I' \rangle \) sharing the same domain \( D \) such that for any \( \Delta \)-model \( M^* = \langle D, I^* \rangle \), if \( I^*(b) = I(b) \) for all \( b \in B \), then \( I^*(a) \neq I'(a) \) for some \( a \in A \) (that is, fixing the phrases in \( B \) in a certain way excludes some interpretation for the phrases in \( A \) that can otherwise be realized). A set of phrases \( A \) is independent of the set of terms \( B \) if it does not depend on it.

**Definition 3** (Determinateness) A phrase \( p \) is determined by the set of phrases \( B \) (w.r.t. \( \Delta \)) if for any two \( \Delta \)-models \( M = \langle D, I \rangle \) and \( M' = \langle D', I' \rangle \), if \( I(b) = I'(b) \) for all \( b \in B \) then \( I(a) = I'(a) \).

Assume now that \( \Delta \) includes the constraint \( I(D) = D \), where \( D \) is a phrase added to \( L \), which is constrained in each model to denote the domain. We can define logical terms as a special case in the system:

**Definition 4** (Logical term) A term \( a \) is a logical term (w.r.t. \( \Delta \)) if it is determined (w.r.t. \( \Delta \)) by the domain, i.e. by \( \langle D \rangle \).

\( ^2 \)The notion of determined by is the same as that of implicitly defined by on some definitions of that term.
The proposed definition of independence diverges from ordinary ones, where independence is taken as the contrary of determinacy (e.g. (Tarski, 1934)). An advantageous result is that logical truths, despite being entailed by any sentence, are independent of all other sentences (compare e.g. (Zimmermann, 1999).

We have the following result:

**Proposition 1**  For every term $a$ and set of terms $B$:

1. If $a$ is a logical term, then $a$ is independent of $B$.
2. If $a$ is determined by $B$, and $a$ is not a logical term, then $a$ depends on $B$.

I propose taking a set of constraints as determining the forms of sentences in the language. This may seem to defy a longstanding practice in the logical tradition, whereby the form of a sentence is represented by a schema. Based on a newly defined notion of **semantic category**, I define a generalized notion of a schema, and show that schemas so defined are a natural way to represent forms of sentences in the system. I prove that schemas satisfy the requirement that either all their instances are valid or none are. Further notions that are defined in the framework are: **sentences**, **families of dependent terms**, **compositionality**, **extensional language**, a term non-trivially constrained by a semantic constraint relative to a system of constraints, etc.

Finally, I discuss further philosophical aspects and implications of the framework. Semantic constraints can be perceived as commitments with respect to language made in the context of formal reasoning. I adopt and modify Stewart Shapiro’s **blended approach**, according to which models represent possible worlds under interpretations of the nonlogical vocabulary (Shapiro, 1998). Since the role of logical terms is now fulfilled by semantic constraints, I take models to represent possible worlds under commitments made with respect to language. I mention ways of extending the framework and the philosophical approach to possible worlds models, and compare my approach to models to the one proposed in (Zimmermann, 1999).

**References**

